INTERFEROMETRIC PHASE AND FREQUENCY MEASUREMENTS OF THE PROPAGATION SPEED OF ACOUSTIC WAVES

V. I. Krylovich and An. S. Rubanov

UDC 534.6:534.843.224

It is shown that direct measurements of the phase and frequency of the electrical signal at the interferometer output yield an increase in the sensitivity and accuracy with which the propagation speed of acoustic waves in a medium can be determined.

Acoustical interferometry methods are now coming into wider use for measuring and monitoring the properties of materials. The most sensitive and accurate of these are methods based on determining the propagation speed and absorption coefficient for acoustic waves in the resonator cavity of an interferometer by measuring the parameters of its amplitude–frequency and phase–frequency curves (see, for example, [1]). However, the capabilities of these methods are limited because of the use of a harmonic input voltage and the requirement that the medium be stationary during the time necessary for measuring these interferometer characteristics. As shown below, the region where acoustic interferometers may be used can be made significantly larger by turning to direct measurements of the phase and frequency of the output electrical signal.

We shall first discuss the case when the homogeneous medium under study filling the resonator cavity of an acoustic interferometer with a harmonic input voltage is in a stationary state. Using the method presented in [2], the following expression can be obtained for the phase $\psi_j(\omega, v)$ of the electrical signal at the interferometer output with one (j = 1) or two (j = 2) flat, linear electroacoustic transducers:

$$\begin{aligned} \mathrm{tg}\,[\psi_j(\omega,\ v) - \omega t - \varphi_0 - \varphi - \varphi_j] = & -\{\exp(-\alpha x_j)\sin(kx_j) + \\ & + r_2\exp[-\alpha(2L - x_j)]\sin[k(2L - x_j) + \delta_2] + R\exp(-\alpha x_j)\times \\ & \times \sin[k(2L - x_j) + \delta_1 + \delta_2] + r_2R\exp[-\alpha(2L - x_j)]\sin(kx_j + \delta_1)\}\times \\ & \times\{\exp(-\alpha x_j)\cos(kx_j) + r_2\exp[-\alpha(2L - x_j)]\cos[k(2L - x_j) + \delta_2] - \\ & - R\exp(-\alpha x_j)\cos[k(2L - x_j) + \delta_1 + \delta_2] - \\ & - r_2R\exp[-\alpha(2L - x_j)]\cos(kx_j + \delta_1)\}^{-1},\quad x_1 = 0,\quad x_2 = L. \end{aligned} \tag{1}$$

If the propagation speed of the waves in the resonator cavity increases by the differential Δv_0, the phase $\psi_j(\omega, v_0)$ changes by $\Delta\psi_j(\omega, v_0, \Delta v_0)$. Obviously, if the quantities Δv_0 and $\Delta\psi_j(\omega, v_0, \Delta v_0)$ are small, the relationship between them can be described by the equation

$$\Delta\psi_j(\omega,\ v_0,\ \Delta v_0) = \frac{\partial\psi_j(\omega,\ v_0)}{\partial v}\ \Delta v_0, \tag{2}$$

where [see Eq. (1)]

$$\begin{aligned} \frac{\partial\psi_j(\omega,\ v)}{\partial v} = & -\{\{\exp[2\alpha(L - x_j)] - r_2^2R^2\exp[-2\alpha(L - x_j)]\}\,x_j + \\ & + \{r_2^2\exp[-2\alpha(L - x_j)] - R^2\exp[2\alpha(L - x_j)]\}(2L - x_j) + \\ & + 2R\{\exp[2\alpha(L - x_j)] - r_2^2\exp[-2\alpha(L - x_j)]\}(L - x_j)\times \end{aligned}$$

Institute for Applied Physics, Academy of Sciences of the Belorussian SSR. Belorussian Institute for the Mechanization of Agriculture, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 4, pp. 654-658, October, 1985. Original article submitted September 9, 1984.

0022-0841/85/4904-1225$09.50

$$\times \cos(2kL + \delta_1 + \delta_2) + 2Lr_2(1 - R^2)\cos[2k(L - x_j) + \delta_2]\} \times \\ \times [1 - 2R\cos(2kL + \delta_1 + \delta_2) + R^2]^{-1}\{\exp[2\alpha(L - x_j)] + \\ + 2r_2\cos[2k(L - x_j) + \delta_2] + r_2^2\exp[-2\alpha(L - x_j)]\}^{-1}\frac{\omega}{v^2}. \tag{3}$$

Starting from Eq. (1), it can be shown that function (3) is related to the slope $\partial\psi_j(\omega, v)/\partial\omega$ of the phase–frequency curve (PFC) in the following way:

$$\frac{\partial\psi_j(\omega, v)}{\partial v} = -\frac{\partial\psi_j(\omega, v)}{\partial\omega}\frac{\omega}{v}[1 + F(\omega, v)], \tag{4}$$

where $|F(\omega, v)|$ is, as a rule, much less than unity for weakly absorbing media. For example,

$$F(\omega, v) = -\frac{4R\alpha v}{\omega}\sin\gamma(\omega, v)$$

for a two-transducer interferometer. It is obvious from the latter expression that, for media similar to water, $|F(\omega, v)| \ll 2\cdot10^{-5}$ at a frequency $f \sim 1$ MHz.

Using (3) and (4) with $|F(\omega, v)| \ll 1$, Eq. (2) takes the form

$$\Delta\psi_j(\omega, v_0, \Delta v_0) = -\frac{\partial\psi_j(\omega, v_0)}{\partial\omega}\frac{\omega}{v_0}\Delta v_0. \tag{5}$$

Equation (5) can be used to determine the size of sufficiently small ($\omega L|\Delta v_0|v_0^{-2} \ll \pi$) differentials in the propagation speed Δv_0 of the acoustic waves from the change in the phase of the interferometer output signal, $\Delta\psi_j(\omega, v_0, \Delta v_0)$, and the slope of its PFC $\partial\omega_j(\omega, v_0)/\partial\omega$. Since the slope of the PFC is maximum for values of L and ω satisfying the condition $\gamma(\omega, v_0) = \gamma(\omega_n, v_0) = 2\pi n$ ($n = 1, 2, \ldots$), the sensitivity of the phase $\psi_j(\omega_n, v_0)$ of the output signal to a change in velocity will be the highest in this case. Thus, from Eqs. (3) and (5) with $\gamma(\omega, v_0) = \gamma(\omega_n, v_0)$, we find that

$$\Delta\psi_2(\omega_n, v_0, \Delta v_0) = -\frac{1 + R}{1 - R}\frac{\omega_n L}{v_0^2}\Delta v_0$$

for a two-transducer interferometer; from this, it follows that the sensitivity with which Δv_0 can be determined by the phase interferometric measurements is a factor of $(1 + R)/(1 - R)$ greater than that for the traveling wave mode ($R = 0$) at the same frequency with the same measuring system baseline. For example, with $f = 1$ MHz, $r_1 = r_2 = 1$, $L = 10^{-2}$ m, $\alpha = 2.3\cdot10^{-2}$ m^{-1} (for water) then $(1 + R)/(1 - R) \approx 4\cdot10^3$.

The method for determining small differentials in the propagation speed of acoustic waves based on direct interferometric phase measurements can also be used for a nonstationary medium.

Suppose that the propagation velocity of the waves in the interferometer resonator cavity varies with time according to the linear law $v(t) = v_0 + bt$. It can then be shown that for a sufficiently small value of $|b|$ ($|b|t \ll v_0$ and $\omega L|b|tv_0^{-2} \ll \pi$ over the entire duration of the measurements), the differential change in the phase of the electrical signal, $\Delta\psi_j(\omega, v(t) - v_0)$, will be related to the velocity $v(t)$ in the following way (as long as $|\Delta\psi_j(\omega, v(t) - v_0)| \ll \pi$):

$$\Delta\psi_j(\omega, v(t) - v_0) = -\frac{\partial\psi_j(\omega, \omega_0)}{\partial\omega}\frac{\omega}{v_0}[v(t) - v_0], \tag{6}$$

and the value of the parameter b will determine the value of the frequency of the electrical signal ω' at the transducer output:

$$\omega' = \omega_0 - \frac{\partial\psi_j(\omega, v_0)}{\partial\omega}\frac{\omega}{v_0}b. \tag{7}$$

From Eqs. (6) and (7), it is evident that it is possible to determine both the instantaneous value of the velocity $v(t)$ and the parameter b which determines its rate of change, knowing the initial velocity v_0, the frequency of the input voltage ω, and the slope of the interferometer PFC $\partial\psi_j(\omega, v_0)/\partial\omega$.

Obviously, if the variation in the parameter $b = b(t)$ is sufficiently slow (i.e., $|db(t)/dt|\tau \ll |b(t)|$, where $\tau = 2L/v_0(1 - R^2)$ is the time for which the acoustic impulse exists in the resonator cavity), the relation between the output frequency $\omega'(t)$ and the parameter b is (under the conditions formulated above) given by an expression analogous to Eq. (7), and it is sufficient to determine the phase differential $\Delta\psi_j(\omega, v(t) - v_0)$ by integrating the frequency shift ($\Delta\psi_j(\omega, v(t) - v_0) = \int_0^t \Delta\omega'(t')dt'$) and then use Eq. (6) to determine the velocity v(t).

The possibility of using direct interferometric phase and frequency measurements to obtain information on the acoustical parameters of the medium with a harmonic probe signal pointed out above indicates that broadening the frequency spectrum of the interferometer input voltage leads to an increase in the information content of the measurements. We shall demonstrate this using an acoustic interferometer with two linear transducers as an example.

Suppose that a harmonic, frequency-modulated voltage of the form

$$U = u_0 \exp\left[-i\left(\omega_0 t + \frac{\Delta\omega_0}{\Omega}\sin\Omega t\right)\right] \quad (8)$$

in complex notation is fed to the input of such an interferometer.

Upon using the Fourier–Bessel series expansion [3] of function (8) and the method in [2], the following expression describing the voltage U' at the interferometer output can be obtained:

$$U' = U_0' \exp\left[-i\omega_0\left(t - \frac{L}{v_0}\right)\right] \sum_{l=-\infty}^{\infty} \frac{J_l\left(\frac{\Delta\omega_0}{\Omega}\right)\exp\left[-il\Omega\left(t - \frac{L}{v_0}\right)\right]}{1 - R\exp\left\{i\left[\frac{2l\Omega L}{v_0} - \gamma(\omega_0, v_0)\right]\right\}}, \quad (9)$$

where U_0' is a constant which is independent of time. As is evident from Eq. (9), the amplitude and frequency of the output signal are, in general, oscillatory functions of the frequency deviation $\Delta\omega_0$ and the modulation frequency Ω of the input voltage, and the form of these functions is determined by the value of the parameter $z = \Omega L/v_0$.

The case where

$$z = z_m = \frac{\pi}{2}(1 + 4m) \quad (m = 0, 1, 2, \ldots) \quad (10)$$

is of the greatest interest.

If the frequency deviation of the input voltage $\Delta\omega_0$ is sufficiently small ($\Delta\omega_0 \ll \Omega$), the following expressions for the amplitude u_0' and frequency $\omega'(t)$ of the electrical signal at the output of the interferometer transducer can be obtained from Eq. (9) provided that condition (10) is satisfied and $\gamma(\omega_0, v_0) = \gamma(\omega_p, v_0) = (2p + 1)\pi$ (p = 0, 1, 2, ...):

$$u_0' = \frac{|U_0'|}{1 + R}, \quad (11)$$

$$\omega'(t) = \omega_p + \frac{1 + R}{1 - R}\Delta\omega_0 \sin\Omega t. \quad (12)$$

It follows from (11) and (12) that under the conditions mentioned above there is no amplitude modulation of the interferometer output voltage, and the frequency deviation of this voltage is a factor of (1 + R)/(1 − R) greater than the input.

If the modulation frequency of the input voltage corresponds to a z-value such that $z - z_m = \nu$, $|\nu| \ll \pi$, amplitude modulation in the interferometer output voltage sets in at a frequency equal to the initial input-signal modulation frequency:

$$u_0'(t, \nu) = \frac{|U_0'|}{1 + R}\left[1 + \nu\frac{\Delta\omega_0}{\Omega}\frac{4R(1 + R^2)}{(1 - R)^3(1 + R)}\sin\Omega t\right]. \quad (13)$$

In this case, the output-voltage frequency is given by the function

$$\omega'(t, \nu) = \omega_p + \frac{1 + R}{1 - R}\Delta\omega_0\left(\sin\Omega t + \nu\frac{1 + R}{1 - R}\cos\Omega t\right). \quad (14)$$

Taking Eq. (13) into account, we obtain

$$\eta(\nu) = 8|\nu| \frac{\Delta\omega_0}{\Omega} \frac{R(1+R^2)}{(1-R)^3(1+R)} \qquad (15)$$

for the amplitude modulation parameter

$$\eta(\nu) = \frac{|u_0'(t, \nu)|_{\max} - |u_0'(t, \nu)|_{\min}}{u_0'(t, 0)}.$$

Introducing the frequency-modulation parameter

$$\mu(\nu) = \frac{\omega'_{\max}(t, \nu) - \omega'_{\min}(t, \nu)}{\omega_p},$$

the following expression may be obtained from Eq. (14):

$$\mu(\nu) = 2|\nu| \frac{\Delta\omega_0}{\omega_p} \left(\frac{1+R}{1-R}\right)^2. \qquad (16)$$

It follows from Eqs. (15) and (16) that the derivatives $d\eta(\nu)/d\nu$ and $d\mu(\nu)/d\nu$ undergo a discontinuity at the point $\nu = 0$, which indicates that there is a sharp change in the amplitude-modulation and frequency-modulation parameters as the frequency Ω is adjusted away from the value satisfying Eq. (10). This circumstance can be used to determine the absolute value of the propagation velocity for the acoustic waves. In fact, by varying the input-voltage modulation frequency from zero to the value $\Omega = \Omega_m$ satisfying condition (10), it is possible to determine the lag time $\tau = L/v_0$ from the relation $\Omega_m L/v_0 = \pi(1 + 4m)/2$, and then the velocity at which the waves propagate, v_0.

Adding the fact that parameters (15) and (16) describing the amplitude and frequency modulation of the interferometer-output voltage are equal to zero, the value of the frequency Ω_m and therefore the propagation velocity of the waves v_0 can also be determined to high accuracy.

NOTATION

u_0, ω_0, $\Delta\omega_0$, Ω, and φ_0, amplitude, carrier frequency, frequency deviation, modulation frequency, and the initial phase of the input voltage, respectively; φ and φ_j, phase lags in the emitting and receiving transducers; α and v, absorption coefficient and acoustic-wave propagation velocity in the interferometer resonator cavity; L, length of the resonator cavity; $k = \omega/v$; $R = r_1 r_2 \exp(-2\alpha L)$; $r_1 \exp(i\delta_1)$ and $r_2 \exp(i\delta_2)$, complex coefficients of reflection of waves from the first (emitting) and second mirrors, respectively; $\gamma(\omega, v) = 2\omega L/v + \delta_1 + \delta_2$; $f = \omega/2\pi$; u_0' and ω', amplitude and frequency of the output voltage; and $J_l(\Delta\omega_0/\Omega)$, Bessel function of the first kind of order l; $i^2 = -1$.

LITERATURE CITED

1. A. P. Sarvasyan, "Development of methods of precise ultrasonic measurements in small volume of liquids," Ultrasonics, 20, No. 4, 151-154 (1982).
2. V. I. Krylovich, P. N. Logvinovich, and A. S. Rubanov, "Taking the influence of reverberation interference into account when studying and monitoring physical processes using the nonstationary acoustic Doppler effect," Vestsi Akad. Navuk BSSR, Ser. Fiz. Tekh. Navuk, No. 3, 100-103 (1979).
3. A. A. Kharkevich, Spectra and Analysis [in Russian], Fizmatgiz, Moscow (1962).